

# Veriopt Theories

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## 1 Conditional Elimination Phase

This theory presents the specification of the `ConditionalElimination` phase within the GraalVM compiler. The `ConditionalElimination` phase simplifies any condition of an *if* statement that can be implied by the conditions that dominate it. Such that if condition A implies that condition B *must* be true, the condition B is simplified to `true`.

```
if (A) {  
  if (B) {  
    ...  
  }  
}
```

We begin by defining the individual implication rules used by the phase in 1.1. These rules are then lifted to the rewriting of a condition within an *if* statement in ???. The traversal algorithm used by the compiler is specified in ???.

```
theory ConditionalElimination  
  imports  
    Semantics.IRTreeEvalThms  
    Proofs.Rewrites  
    Proofs.Bisimulation  
    OptimizationDSL.Markup  
begin
```

**declare**  $[[show-types=false]]$

## 1.1 Implication Rules

The set of rules used for determining whether a condition,  $q_1$ , implies another condition,  $q_2$ , must be true or false.

### 1.1.1 Structural Implication

The first method for determining if a condition can be implied by another condition, is structural implication. That is, by looking at the structure of the conditions, we can determine the truth value. For instance,  $x \equiv y$  implies that  $x < y$  cannot be true.

**inductive**

*impliesx* ::  $IRExpr \Rightarrow IRExpr \Rightarrow bool$  ( $- \Rightarrow -$ ) **and**  
*impliesnot* ::  $IRExpr \Rightarrow IRExpr \Rightarrow bool$  ( $- \Rightarrow \neg -$ ) **where**  
*same*:  $q \Rightarrow q$  |  
*eq-not-less*:  $exp[x \text{ eq } y] \Rightarrow \neg exp[x < y]$  |  
*eq-not-less'*:  $exp[x \text{ eq } y] \Rightarrow \neg exp[y < x]$  |  
*less-not-less*:  $exp[x < y] \Rightarrow \neg exp[y < x]$  |  
*less-not-eq*:  $exp[x < y] \Rightarrow \neg exp[x \text{ eq } y]$  |  
*less-not-eq'*:  $exp[x < y] \Rightarrow \neg exp[y \text{ eq } x]$  |  
*negate-true*:  $[[x \Rightarrow \neg y]] \Longrightarrow x \Rightarrow exp[!y]$  |  
*negate-false*:  $[[x \Rightarrow y]] \Longrightarrow x \Rightarrow \neg exp[!y]$

**inductive** *implies-complete* ::  $IRExpr \Rightarrow IRExpr \Rightarrow bool \text{ option} \Rightarrow bool$  **where**

*implies*:  
 $x \Rightarrow y \Longrightarrow implies-complete\ x\ y\ (Some\ True)$  |  
*impliesnot*:  
 $x \Rightarrow \neg y \Longrightarrow implies-complete\ x\ y\ (Some\ False)$  |  
*fail*:  
 $\neg((x \Rightarrow y) \vee (x \Rightarrow \neg y)) \Longrightarrow implies-complete\ x\ y\ None$

The relation  $q_1 \Rightarrow q_2$  requires that the implication  $q_1 \longrightarrow q_2$  is known true (i.e. universally valid). The relation  $q_1 \Rightarrow \neg q_2$  requires that the implication  $q_1 \longrightarrow q_2$  is known false (i.e.  $q_1 \longrightarrow \neg q_2$  is universally valid). If neither  $q_1 \Rightarrow q_2$  nor  $q_1 \Rightarrow \neg q_2$  then the status is unknown and the condition cannot be simplified.

**fun** *implies-valid* ::  $IRExpr \Rightarrow IRExpr \Rightarrow bool$  (**infix**  $\rightsquigarrow 50$ ) **where**

*implies-valid*  $q1\ q2 =$   
 $(\forall m\ p\ v1\ v2. ([m, p] \vdash q1 \mapsto v1) \wedge ([m, p] \vdash q2 \mapsto v2) \longrightarrow$   
 $(val-to-bool\ v1 \longrightarrow val-to-bool\ v2))$

**fun** *impliesnot-valid* ::  $IRExpr \Rightarrow IRExpr \Rightarrow bool$  (**infix**  $\rightsquigarrow 50$ ) **where**

*impliesnot-valid*  $q1\ q2 =$   
 $(\forall m\ p\ v1\ v2. ([m, p] \vdash q1 \mapsto v1) \wedge ([m, p] \vdash q2 \mapsto v2) \longrightarrow$

$$(val\text{-to-bool } v1 \longrightarrow \neg val\text{-to-bool } v2))$$

The relation  $q_1 \rightsquigarrow q_2$  means  $q_1 \longrightarrow q_2$  is universally valid, and the relation  $q_1 \rightsquigarrow\!\!\!\rightarrow q_2$  means  $q_1 \longrightarrow \neg q_2$  is universally valid.

**lemma** *eq-not-less-val:*

$$val\text{-to-bool}(val[v1 \text{ eq } v2]) \longrightarrow \neg val\text{-to-bool}(val[v1 < v2])$$

*<proof>*

**lemma** *eq-not-less'-val:*

$$val\text{-to-bool}(val[v1 \text{ eq } v2]) \longrightarrow \neg val\text{-to-bool}(val[v2 < v1])$$

*<proof>*

**lemma** *less-not-less-val:*

$$val\text{-to-bool}(val[v1 < v2]) \longrightarrow \neg val\text{-to-bool}(val[v2 < v1])$$

*<proof>*

**lemma** *less-not-eq-val:*

$$val\text{-to-bool}(val[v1 < v2]) \longrightarrow \neg val\text{-to-bool}(val[v1 \text{ eq } v2])$$

*<proof>*

**lemma** *logic-negate-type:*

$$\text{assumes } [m, p] \vdash \text{UnaryExpr UnaryLogicNegation } x \mapsto v$$

$$\text{shows } \exists b \ v2. [m, p] \vdash x \mapsto \text{IntVal } b \ v2$$

*<proof>*

**lemma** *intval-logic-negation-inverse:*

$$\text{assumes } b > 0$$

$$\text{assumes } x = \text{IntVal } b \ v$$

$$\text{shows } val\text{-to-bool } (\text{intval-logic-negation } x) \longleftrightarrow \neg(val\text{-to-bool } x)$$

*<proof>*

**lemma** *logic-negation-relation-tree:*

$$\text{assumes } [m, p] \vdash y \mapsto val$$

$$\text{assumes } [m, p] \vdash \text{UnaryExpr UnaryLogicNegation } y \mapsto \text{invval}$$

$$\text{shows } val\text{-to-bool } val \longleftrightarrow \neg(val\text{-to-bool } \text{invval})$$

*<proof>*

The following theorem show that the known true/false rules are valid.

**theorem** *implies-impliesnot-valid:*

$$\text{shows } ((q1 \Rightarrow q2) \longrightarrow (q1 \rightsquigarrow q2)) \wedge$$

$$((q1 \Rightarrow \neg q2) \longrightarrow (q1 \rightsquigarrow\!\!\!\rightarrow q2))$$

$$(\text{is } (?imp \longrightarrow ?val) \wedge (?notimp \longrightarrow ?notval))$$

*<proof>*

### 1.1.2 Type Implication

The second mechanism to determine whether a condition implies another is to use the type information of the relevant nodes. For instance,  $x < (4 :: 'a)$

implies  $x < (10::'a)$ . We can show this by strengthening the type, stamp, of the node  $x$  such that the upper bound is  $4::'a$ . Then we the second condition is reached, we know that the condition must be true by the upperbound.

The following relation corresponds to the `UnaryOpLogicNode.tryFold` and `BinaryOpLogicNode.tryFold` methods and their associated concrete implementations.

We track the refined stamps by mapping nodes to Stamps, the second parameter to `tryFold`.

```
inductive tryFold :: IRNode ⇒ (ID ⇒ Stamp) ⇒ bool ⇒ bool
where
  [[alwaysDistinct (stamps x) (stamps y)]]
    ⇒ tryFold (IntegerEqualsNode x y) stamps False |
  [[neverDistinct (stamps x) (stamps y)]]
    ⇒ tryFold (IntegerEqualsNode x y) stamps True |
  [[is-IntegerStamp (stamps x);
    is-IntegerStamp (stamps y);
    stpi-upper (stamps x) < stpi-lower (stamps y)]]
    ⇒ tryFold (IntegerLessThanNode x y) stamps True |
  [[is-IntegerStamp (stamps x);
    is-IntegerStamp (stamps y);
    stpi-lower (stamps x) ≥ stpi-upper (stamps y)]]
    ⇒ tryFold (IntegerLessThanNode x y) stamps False
```

```
code-pred (modes: i ⇒ i ⇒ i ⇒ bool) tryFold ⟨proof⟩
```

Prove that, when the stamp map is valid, the `tryFold` relation correctly predicts the output value with respect to our evaluation semantics.

**inductive-cases** *StepE*:

```
g, p ⊢ (nid,m,h) → (nid',m',h)
```

**lemma** *is-stamp-empty-valid*:

```
assumes is-stamp-empty s
shows ¬(∃ val. valid-value val s)
⟨proof⟩
```

**lemma** *join-valid*:

```
assumes is-IntegerStamp s1 ∧ is-IntegerStamp s2
assumes valid-stamp s1 ∧ valid-stamp s2
shows (valid-value v s1 ∧ valid-value v s2) = valid-value v (join s1 s2) (is ?lhs
= ?rhs)
⟨proof⟩
```

**lemma** *alwaysDistinct-evaluate*:

```
assumes wf-stamp g stamps
assumes alwaysDistinct (stamps x) (stamps y)
```

**assumes** *is-IntegerStamp* (stamps x)  $\wedge$  *is-IntegerStamp* (stamps y)  $\wedge$  *valid-stamp*  
 (stamps x)  $\wedge$  *valid-stamp* (stamps y)  
**shows**  $\neg(\exists \text{ val} . ([g, m, p] \vdash x \mapsto \text{val}) \wedge ([g, m, p] \vdash y \mapsto \text{val}))$   
 <proof>

**lemma** *alwaysDistinct-valid*:

**assumes** *wf-stamp* g stamps  
**assumes** *kind* g nid = (*IntegerEqualsNode* x y)  
**assumes** [g, m, p]  $\vdash$  nid  $\mapsto$  v  
**assumes** *alwaysDistinct* (stamps x) (stamps y)  
**shows**  $\neg(\text{val-to-bool } v)$   
 <proof>

**thm-oracles** *alwaysDistinct-valid*

**lemma** *unwrap-valid*:

**assumes**  $0 < b \wedge b \leq 64$   
**assumes** *take-bit* (b::nat) (vv::64 word) = vv  
**shows** (vv::64 word) = *take-bit* b (*word-of-int* (*int-signed-value* (b::nat) (vv::64  
 word)))  
 <proof>

**lemma** *asConstant-valid*:

**assumes** *asConstant* s = val  
**assumes** val  $\neq$  *UndefVal*  
**assumes** *valid-value* v s  
**shows** v = val  
 <proof>

**lemma** *neverDistinct-valid*:

**assumes** *wf-stamp* g stamps  
**assumes** *kind* g nid = (*IntegerEqualsNode* x y)  
**assumes** [g, m, p]  $\vdash$  nid  $\mapsto$  v  
**assumes** *neverDistinct* (stamps x) (stamps y)  
**shows** *val-to-bool* v  
 <proof>

**lemma** *stampUnder-valid*:

**assumes** *wf-stamp* g stamps  
**assumes** *kind* g nid = (*IntegerLessThanNode* x y)  
**assumes** [g, m, p]  $\vdash$  nid  $\mapsto$  v  
**assumes** *stpi-upper* (stamps x) < *stpi-lower* (stamps y)  
**shows** *val-to-bool* v  
 <proof>

**lemma** *stampOver-valid*:

**assumes** *wf-stamp* g stamps  
**assumes** *kind* g nid = (*IntegerLessThanNode* x y)  
**assumes** [g, m, p]  $\vdash$  nid  $\mapsto$  v  
**assumes** *stpi-lower* (stamps x)  $\geq$  *stpi-upper* (stamps y)

**shows**  $\neg(\text{val-to-bool } v)$   
 $\langle \text{proof} \rangle$

**theorem** *tryFoldTrue-valid*:  
**assumes** *wf-stamp g stamps*  
**assumes** *tryFold (kind g nid) stamps True*  
**assumes**  $[g, m, p] \vdash \text{nid} \mapsto v$   
**shows** *val-to-bool v*  
 $\langle \text{proof} \rangle$

**theorem** *tryFoldFalse-valid*:  
**assumes** *wf-stamp g stamps*  
**assumes** *tryFold (kind g nid) stamps False*  
**assumes**  $[g, m, p] \vdash \text{nid} \mapsto v$   
**shows**  $\neg(\text{val-to-bool } v)$   
 $\langle \text{proof} \rangle$

## 1.2 Lift rules

**inductive** *condset-implies* ::  $IRExpr \text{ set} \Rightarrow IRExpr \Rightarrow \text{bool} \Rightarrow \text{bool}$  **where**  
*impliesTrue*:  
 $(\exists ce \in \text{conds} . (ce \Rightarrow \text{cond})) \Longrightarrow \text{condset-implies } \text{conds } \text{cond } \text{True} \mid$   
*impliesFalse*:  
 $(\exists ce \in \text{conds} . (ce \Rightarrow \neg \text{cond})) \Longrightarrow \text{condset-implies } \text{conds } \text{cond } \text{False}$

**code-pred** (*modes: i  $\Rightarrow$  i  $\Rightarrow$  i  $\Rightarrow$  bool*) *condset-implies*  $\langle \text{proof} \rangle$

The *cond-implies* function lifts the structural and type implication rules to the one relation.

**fun** *conds-implies* ::  $IRExpr \text{ set} \Rightarrow (ID \Rightarrow \text{Stamp}) \Rightarrow IRNode \Rightarrow IRExpr \Rightarrow \text{bool option}$  **where**  
*conds-implies* *conds stamps condNode cond* =  
*if condset-implies* *conds cond True*  $\vee$  *tryFold condNode stamps True*  
*then Some True*  
*else if condset-implies* *conds cond False*  $\vee$  *tryFold condNode stamps False*  
*then Some False*  
*else None*)

Perform conditional elimination rewrites on the graph for a particular node by lifting the individual implication rules to a relation that rewrites the condition of *if* statements to constant values.

In order to determine conditional eliminations appropriately the rule needs two data structures produced by static analysis. The first parameter is the set of IRNodes that we know result in a true value when evaluated. The second parameter is a mapping from node identifiers to the flow-sensitive stamp.

**inductive** *ConditionalEliminationStep* ::  
 $IRExpr \text{ set} \Rightarrow (ID \Rightarrow \text{Stamp}) \Rightarrow ID \Rightarrow IRGraph \Rightarrow IRGraph \Rightarrow \text{bool}$

**where**

*impliesTrue:*

```
[[kind g ifcond = (IfNode cid t f);  
  g ⊢ cid ≃ cond;  
  condNode = kind g cid;  
  conds-implies conds stamps condNode cond = (Some True);  
  g' = constantCondition True ifcond (kind g ifcond) g  
]] ⇒ ConditionalEliminationStep conds stamps ifcond g g' |
```

*impliesFalse:*

```
[[kind g ifcond = (IfNode cid t f);  
  g ⊢ cid ≃ cond;  
  condNode = kind g cid;  
  conds-implies conds stamps condNode cond = (Some False);  
  g' = constantCondition False ifcond (kind g ifcond) g  
]] ⇒ ConditionalEliminationStep conds stamps ifcond g g' |
```

*unknown:*

```
[[kind g ifcond = (IfNode cid t f);  
  g ⊢ cid ≃ cond;  
  condNode = kind g cid;  
  conds-implies conds stamps condNode cond = None  
]] ⇒ ConditionalEliminationStep conds stamps ifcond g g |
```

*notIfNode:*

```
¬(is-IfNode (kind g ifcond)) ⇒  
  ConditionalEliminationStep conds stamps ifcond g g
```

**code-pred** (*modes*:  $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow \text{bool}$ ) *ConditionalEliminationStep*  
{*proof*}

**thm** *ConditionalEliminationStep.equation*

### 1.3 Control-flow Graph Traversal

**type-synonym** *Seen* = *ID set*

**type-synonym** *Condition* = *IRExpr*

**type-synonym** *Conditions* = *Condition list*

**type-synonym** *StampFlow* = (*ID* ⇒ *Stamp*) *list*

**type-synonym** *ToVisit* = *ID list*

*nextEdge* helps determine which node to traverse next by returning the first successor edge that isn't in the set of already visited nodes. If there is not an appropriate successor, *None* is returned instead.

**fun** *nextEdge* :: *Seen* ⇒ *ID* ⇒ *IRGraph* ⇒ *ID option* **where**

*nextEdge* *seen* *nid* *g* =

```
(let nids = (filter (λnid'. nid' ∉ seen) (successors-of (kind g nid))) in  
  (if length nids > 0 then Some (hd nids) else None))
```

*pred* determines which node, if any, acts as the predecessor of another.

Merge nodes represent a special case wherein the predecessor exists as an input edge of the merge node, to simplify the traversal we treat only the first input end node as the predecessor, ignoring that multiple nodes may act as a successor.

For all other nodes, the predecessor is the first element of the predecessors set. Note that in a well-formed graph there should only be one element in the predecessor set.

```
fun preds :: IRGraph ⇒ ID ⇒ ID list where
  preds g nid = (case kind g nid of
    (MergeNode ends -) ⇒ ends |
    - ⇒
      sorted-list-of-set (IRGraph.predecessors g nid)
  )
```

```
fun pred :: IRGraph ⇒ ID ⇒ ID option where
  pred g nid = (case preds g nid of [] ⇒ None | x # xs ⇒ Some x)
```

When the basic block of an if statement is entered, we know that the condition of the preceding if statement must be true. As in the GraalVM compiler, we introduce the `registerNewCondition` function which roughly corresponds to `ConditionalEliminationPhase.registerNewCondition`. This method updates the flow-sensitive stamp information based on the condition which we know must be true.

```
fun clip-upper :: Stamp ⇒ int ⇒ Stamp where
  clip-upper (IntegerStamp b l h) c =
    (if c < h then (IntegerStamp b l c) else (IntegerStamp b l h)) |
  clip-upper s c = s
```

```
fun clip-lower :: Stamp ⇒ int ⇒ Stamp where
  clip-lower (IntegerStamp b l h) c =
    (if l < c then (IntegerStamp b c h) else (IntegerStamp b l c)) |
  clip-lower s c = s
```

```
fun max-lower :: Stamp ⇒ Stamp ⇒ Stamp where
  max-lower (IntegerStamp b1 xl xh) (IntegerStamp b2 yl yh) =
    (IntegerStamp b1 (max xl yl) xh) |
  max-lower xs ys = xs
```

```
fun min-higher :: Stamp ⇒ Stamp ⇒ Stamp where
  min-higher (IntegerStamp b1 xl xh) (IntegerStamp b2 yl yh) =
    (IntegerStamp b1 yl (min xh yh)) |
  min-higher xs ys = ys
```

```
fun registerNewCondition :: IRGraph ⇒ IRNode ⇒ (ID ⇒ Stamp) ⇒ (ID ⇒ Stamp) where
  — constrain equality by joining the stamps
  registerNewCondition g (IntegerEqualsNode x y) stamps =
    (stamps
```



```

    (x := join (stamps x) (stamps y)))
    (y := join (stamps x) (stamps y)) |
  — constrain less than by removing overlapping stamps
  registerNewCondition g (IntegerLessThanNode x y) stamps =
    (stamps
     (x := clip-upper (stamps x) ((stpi-lower (stamps y)) - 1)))
     (y := clip-lower (stamps y) ((stpi-upper (stamps x)) + 1)) |
  registerNewCondition g (LogicNegationNode c) stamps =
    (case (kind g c) of
     (IntegerLessThanNode x y) =>
      (stamps
       (x := max-lower (stamps x) (stamps y)))
       (y := min-higher (stamps x) (stamps y))
      | - => stamps) |
  registerNewCondition g - stamps = stamps

```

```

fun hdOr :: 'a list => 'a => 'a where
  hdOr (x # xs) de = x |
  hdOr [] de = de

```

**type-synonym** DominatorCache = (ID, ID set) map

**inductive**

```

  dominators-all :: IRGraph => DominatorCache => ID => ID set set => ID list =>
  DominatorCache => ID set set => ID list => bool and
  dominators :: IRGraph => DominatorCache => ID => (ID set × DominatorCache)
=> bool where

```

```

[[pre = []]]
=> dominators-all g c nid doms pre c doms pre |

```

```

[[pre = pr # xs;
 (dominators g c pr (doms', c'));
 dominators-all g c' pr (doms ∪ {doms'}) xs c'' doms'' pre']]
=> dominators-all g c nid doms pre c'' doms'' pre' |

```

```

[[preds g nid = []]]
=> dominators g c nid ({nid}, c) |

```

```

[[c nid = None;
 preds g nid = x # xs;
 dominators-all g c nid {} (preds g nid) c' doms pre';
 c'' = c'(nid ↦ ({nid} ∪ (∩ doms)))]]
=> dominators g c nid (({nid} ∪ (∩ doms)), c'') |

```

```

[[c nid = Some doms]]
=> dominators g c nid (doms, c)

```

— Trying to simplify by removing the 3rd case won't work. A base case for root nodes is required as  $\bigcap \emptyset = \text{coset } []$  which swallows anything unioned with it.

```

value  $\bigcap (\{\}::\text{nat set set})$ 
value  $-\bigcap (\{\}::\text{nat set set})$ 
value  $\bigcap (\{\{\}, \{0\}\}::\text{nat set set})$ 
value  $\{0::\text{nat}\} \cup (\bigcap \{\})$ 

```

```

code-pred (modes:  $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow \text{bool}$ ) dominators-all
<proof>

```

```

code-pred (modes:  $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow \text{bool}$ ) dominators <proof>

```

**definition** *ConditionalEliminationTest13-testSnippet2-initial* :: *IRGraph* where

```

ConditionalEliminationTest13-testSnippet2-initial = irgraph [
  (0, (StartNode (Some 2) 8), VoidStamp),
  (1, (ParameterNode 0), IntegerStamp 32 (-2147483648) (2147483647)),
  (2, (FrameState [] None None None), IllegalStamp),
  (3, (ConstantNode (new-int 32 (0))), IntegerStamp 32 (0) (0)),
  (4, (ConstantNode (new-int 32 (1))), IntegerStamp 32 (1) (1)),
  (5, (IntegerLessThanNode 1 4), VoidStamp),
  (6, (BeginNode 13), VoidStamp),
  (7, (BeginNode 23), VoidStamp),
  (8, (IfNode 5 7 6), VoidStamp),
  (9, (ConstantNode (new-int 32 (-1))), IntegerStamp 32 (-1) (-1)),
  (10, (IntegerEqualsNode 1 9), VoidStamp),
  (11, (BeginNode 17), VoidStamp),
  (12, (BeginNode 15), VoidStamp),
  (13, (IfNode 10 12 11), VoidStamp),
  (14, (ConstantNode (new-int 32 (-2))), IntegerStamp 32 (-2) (-2)),
  (15, (StoreFieldNode 15 "org.graalvm.compiler.core.test.ConditionalEliminationTestBase::sink2"
14 (Some 16) None 19), VoidStamp),
  (16, (FrameState [] None None None), IllegalStamp),
  (17, (EndNode), VoidStamp),
  (18, (MergeNode [17, 19] (Some 20) 21), VoidStamp),
  (19, (EndNode), VoidStamp),
  (20, (FrameState [] None None None), IllegalStamp),
  (21, (StoreFieldNode 21 "org.graalvm.compiler.core.test.ConditionalEliminationTestBase::sink1"
3 (Some 22) None 25), VoidStamp),
  (22, (FrameState [] None None None), IllegalStamp),
  (23, (EndNode), VoidStamp),
  (24, (MergeNode [23, 25] (Some 26) 27), VoidStamp),
  (25, (EndNode), VoidStamp),
  (26, (FrameState [] None None None), IllegalStamp),
  (27, (StoreFieldNode 27 "org.graalvm.compiler.core.test.ConditionalEliminationTestBase::sink0"
9 (Some 28) None 29), VoidStamp),
  (28, (FrameState [] None None None), IllegalStamp),
  (29, (ReturnNode None None), VoidStamp)
]

```

**values**  $\{(snd\ x)\ 13\ |\ x.\ dominators\ ConditionalEliminationTest13-testSnippet2-initial\ Map.empty\ 25\ x\}$

**inductive**

*condition-of* :: *IRGraph*  $\Rightarrow$  *ID*  $\Rightarrow$  (*IRExpr*  $\times$  *IRNode*) *option*  $\Rightarrow$  *bool* **where**  
 $\llbracket$ Some *ifcond* = *pred* *g* *nid*;  
*kind* *g* *ifcond* = *IfNode* *cond* *t* *f*;

*i* = *find-index* *nid* (*successors-of* (*kind* *g* *ifcond*));  
*c* = (*if* *i* = 0 *then* *kind* *g* *cond* *else* *LogicNegationNode* *cond*);  
*rep* *g* *cond* *ce*;  
*ce'* = (*if* *i* = 0 *then* *ce* *else* *UnaryExpr* *UnaryLogicNegation* *ce*)  
 $\Rightarrow$  *condition-of* *g* *nid* (*Some* (*ce'*, *c*)) |

$\llbracket$ *pred* *g* *nid* = *None* $\rrbracket \Rightarrow$  *condition-of* *g* *nid* *None* |  
 $\llbracket$ *pred* *g* *nid* = *Some* *nid'*;  
 $\neg$ (*is-IfNode* (*kind* *g* *nid'*)) $\rrbracket \Rightarrow$  *condition-of* *g* *nid* *None*

**code-pred** (*modes*: *i*  $\Rightarrow$  *i*  $\Rightarrow$  *o*  $\Rightarrow$  *bool*) *condition-of*  $\langle$ *proof* $\rangle$

**fun** *conditions-of-dominators* :: *IRGraph*  $\Rightarrow$  *ID* *list*  $\Rightarrow$  *Conditions*  $\Rightarrow$  *Conditions*  
**where**

*conditions-of-dominators* *g* [] *cds* = *cds* |  
*conditions-of-dominators* *g* (*nid* # *nids*) *cds* =  
(*case* (*Predicate.the* (*condition-of-i-i-o* *g* *nid*)) *of*  
*None*  $\Rightarrow$  *conditions-of-dominators* *g* *nids* *cds* |  
*Some* (*expr*, *-*)  $\Rightarrow$  *conditions-of-dominators* *g* *nids* (*expr* # *cds*))

**fun** *stamps-of-dominators* :: *IRGraph*  $\Rightarrow$  *ID* *list*  $\Rightarrow$  *StampFlow*  $\Rightarrow$  *StampFlow*  
**where**

*stamps-of-dominators* *g* [] *stamps* = *stamps* |  
*stamps-of-dominators* *g* (*nid* # *nids*) *stamps* =  
(*case* (*Predicate.the* (*condition-of-i-i-o* *g* *nid*)) *of*  
*None*  $\Rightarrow$  *stamps-of-dominators* *g* *nids* *stamps* |  
*Some* (*-*, *node*)  $\Rightarrow$  *stamps-of-dominators* *g* *nids*  
(*registerNewCondition* *g* *node* (*hd* *stamps*)) # *stamps*))

**inductive**

*analyse* :: *IRGraph*  $\Rightarrow$  *DominatorCache*  $\Rightarrow$  *ID*  $\Rightarrow$  (*Conditions*  $\times$  *StampFlow*  $\times$  *DominatorCache*)  $\Rightarrow$  *bool* **where**  
 $\llbracket$  *dominators* *g c nid* (*doms*, *c'*);  
*conditions-of-dominators* *g* (*sorted-list-of-set* *doms*)  $\llbracket$  = *conds*;  
*stamps-of-dominators* *g* (*sorted-list-of-set* *doms*) [*stamp* *g*] = *stamps* $\rrbracket$   
 $\implies$  *analyse* *g c nid* (*conds*, *stamps*, *c'*)

**code-pred** (*modes*: *i*  $\Rightarrow$  *i*  $\Rightarrow$  *i*  $\Rightarrow$  *o*  $\Rightarrow$  *bool*) *analyse*  $\langle$ *proof* $\rangle$

**values** {*x*. *dominators* *ConditionalEliminationTest13-testSnippet2-initial* *Map.empty* 13 *x*}

**values** {(*conds*, *stamps*, *c*).

*analyse* *ConditionalEliminationTest13-testSnippet2-initial* *Map.empty* 13 (*conds*, *stamps*, *c*)}

**values** {(*hd* *stamps*) 1 | *conds* *stamps* *c* .

*analyse* *ConditionalEliminationTest13-testSnippet2-initial* *Map.empty* 13 (*conds*, *stamps*, *c*)}

**values** {(*hd* *stamps*) 1 | *conds* *stamps* *c* .

*analyse* *ConditionalEliminationTest13-testSnippet2-initial* *Map.empty* 27 (*conds*, *stamps*, *c*)}

**fun** *next-nid* :: *IRGraph*  $\Rightarrow$  *ID* *set*  $\Rightarrow$  *ID*  $\Rightarrow$  *ID* *option* **where**

*next-nid* *g* *seen* *nid* = (*case* (*kind* *g* *nid*) *of*  
(*EndNode*)  $\Rightarrow$  *Some* (*any-usage* *g* *nid*) |  
-  $\Rightarrow$  *nextEdge* *seen* *nid* *g*)

**inductive** *Step*

:: *IRGraph*  $\Rightarrow$  (*ID*  $\times$  *Seen*)  $\Rightarrow$  (*ID*  $\times$  *Seen*) *option*  $\Rightarrow$  *bool*

**for** *g* **where**

— We can find a successor edge that is not in seen, go there

$\llbracket$  *seen'* = {*nid*}  $\cup$  *seen*;

*Some* *nid'* = *next-nid* *g* *seen'* *nid*;

*nid'*  $\notin$  *seen'* $\rrbracket$

$\implies$  *Step* *g* (*nid*, *seen*) (*Some* (*nid'*, *seen'*)) |

— We cannot find a successor edge that is not in seen, give back None

$\llbracket$  *seen'* = {*nid*}  $\cup$  *seen*;

*None* = *next-nid* *g* *seen'* *nid* $\rrbracket$

$\implies$  *Step* *g* (*nid*, *seen*) *None* |

— We've already seen this node, give back None

$\llbracket$  *seen'* = {*nid*}  $\cup$  *seen*;

*Some* *nid'* = *next-nid* *g* *seen'* *nid*;

*nid'*  $\in$  *seen'* $\rrbracket \implies$  *Step* *g* (*nid*, *seen*) *None*

**code-pred** (*modes*:  $i \Rightarrow i \Rightarrow o \Rightarrow \text{bool}$ ) *Step*  $\langle \text{proof} \rangle$

**fun** *nextNode* :: *IRGraph*  $\Rightarrow$  *Seen*  $\Rightarrow$  (*ID*  $\times$  *Seen*) *option* **where**  
*nextNode* *g* *seen* =  
 (let *toSee* = *sorted-list-of-set* { $n \in \text{ids } g. n \notin \text{seen}$ } in  
 case *toSee* of []  $\Rightarrow$  *None* | ( $x \# xs$ )  $\Rightarrow$  *Some* ( $x, \text{seen} \cup \{x\}$ ))

**values** {*x*. *Step ConditionalEliminationTest13-testSnippet2-initial* (17, {17,11,25,21,18,19,15,12,13,6,29,27},  
*x*)}

The *ConditionalEliminationPhase* relation is responsible for combining the individual traversal steps from the *Step* relation and the optimizations from the *ConditionalEliminationStep* relation to perform a transformation of the whole graph.

**inductive** *ConditionalEliminationPhase*  
 :: (*Seen*  $\times$  *DominatorCache*)  $\Rightarrow$  *IRGraph*  $\Rightarrow$  *IRGraph*  $\Rightarrow$  *bool*  
**where**

— Can do a step and optimise for the current node  
 [[*nextNode* *g* *seen* = *Some* (*nid*, *seen'*);

*analyse* *g* *c* *nid* (*conds*, *flow*, *c'*);  
*ConditionalEliminationStep* (*set* *conds*) (*hd* *flow*) *nid* *g* *g'*;

*ConditionalEliminationPhase* (*seen'*, *c'*) *g'* *g''*]]  
 $\Rightarrow$  *ConditionalEliminationPhase* (*seen*, *c*) *g* *g''* |

[[*nextNode* *g* *seen* = *None*]]  
 $\Rightarrow$  *ConditionalEliminationPhase* (*seen*, *c*) *g* *g*

**code-pred** (*modes*:  $i \Rightarrow i \Rightarrow o \Rightarrow \text{bool}$ ) *ConditionalEliminationPhase*  $\langle \text{proof} \rangle$

**definition** *runConditionalElimination* :: *IRGraph*  $\Rightarrow$  *IRGraph* **where**  
*runConditionalElimination* *g* =  
 (*Predicate.the* (*ConditionalEliminationPhase-i-i-o* ({}, *Map.empty*) *g*))

**values** {(*doms*, *c'*) | *doms* *c'*}.  
*dominators* *ConditionalEliminationTest13-testSnippet2-initial* *Map.empty* 6 (*doms*,  
*c'*)}

**values** {(*conds*, *stamps*, *c*) | *conds* *stamps* *c*}.  
*analyse* *ConditionalEliminationTest13-testSnippet2-initial* *Map.empty* 6 (*conds*, *stamps*,  
*c*)}

**value**  
 (*nextNode*  
*ConditionalEliminationTest13-testSnippet2-initial* {0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100})

**lemma** *IfNodeStepE*:  $g, p \vdash (nid, m, h) \rightarrow (nid', m', h) \implies$   
 $(\bigwedge cond\ tb\ fb\ val.$   
 $\quad kind\ g\ nid = IfNode\ cond\ tb\ fb \implies$   
 $\quad nid' = (if\ val\text{-}to\text{-}bool\ val\ then\ tb\ else\ fb) \implies$   
 $\quad [g, m, p] \vdash cond \mapsto val \implies m' = m)$   
 $\langle proof \rangle$

**lemma** *ifNodeHasCondEvalStutter*:  
**assumes**  $(g\ m\ p\ h \vdash nid \rightsquigarrow nid')$   
**assumes**  $kind\ g\ nid = IfNode\ cond\ t\ f$   
**shows**  $\exists v. ([g, m, p] \vdash cond \mapsto v)$   
 $\langle proof \rangle$

**lemma** *ifNodeHasCondEval*:  
**assumes**  $(g, p \vdash (nid, m, h) \rightarrow (nid', m', h'))$   
**assumes**  $kind\ g\ nid = IfNode\ cond\ t\ f$   
**shows**  $\exists v. ([g, m, p] \vdash cond \mapsto v)$   
 $\langle proof \rangle$

**lemma** *replace-if-t*:  
**assumes**  $kind\ g\ nid = IfNode\ cond\ tb\ fb$   
**assumes**  $[g, m, p] \vdash cond \mapsto bool$   
**assumes**  $val\text{-}to\text{-}bool\ bool$   
**assumes**  $g': g' = replace\text{-}usages\ nid\ tb\ g$   
**shows**  $\exists nid'. (g\ m\ p\ h \vdash nid \rightsquigarrow nid') \iff (g'\ m\ p\ h \vdash nid \rightsquigarrow nid')$   
 $\langle proof \rangle$

**lemma** *replace-if-t-imp*:  
**assumes**  $kind\ g\ nid = IfNode\ cond\ tb\ fb$   
**assumes**  $[g, m, p] \vdash cond \mapsto bool$   
**assumes**  $val\text{-}to\text{-}bool\ bool$   
**assumes**  $g': g' = replace\text{-}usages\ nid\ tb\ g$   
**shows**  $\exists nid'. (g\ m\ p\ h \vdash nid \rightsquigarrow nid') \implies (g'\ m\ p\ h \vdash nid \rightsquigarrow nid')$   
 $\langle proof \rangle$

**lemma** *replace-if-f*:  
**assumes**  $kind\ g\ nid = IfNode\ cond\ tb\ fb$   
**assumes**  $[g, m, p] \vdash cond \mapsto bool$   
**assumes**  $\neg(val\text{-}to\text{-}bool\ bool)$   
**assumes**  $g': g' = replace\text{-}usages\ nid\ fb\ g$   
**shows**  $\exists nid'. (g\ m\ p\ h \vdash nid \rightsquigarrow nid') \iff (g'\ m\ p\ h \vdash nid \rightsquigarrow nid')$   
 $\langle proof \rangle$

Prove that the individual conditional elimination rules are correct with respect to preservation of stuttering steps.

**lemma** *ConditionalEliminationStepProof*:

**assumes**  $wg$ : *wf-graph*  $g$   
**assumes**  $ws$ : *wf-stamps*  $g$   
**assumes**  $wv$ : *wf-values*  $g$   
**assumes**  $nid$ :  $nid \in ids\ g$   
**assumes**  $conds$ -*valid*:  $\forall c \in conds . \exists v. ([m, p] \vdash c \mapsto v) \wedge val\text{-to}\text{-bool}\ v$   
**assumes**  $ce$ : *ConditionalEliminationStep*  $conds\ stamps\ nid\ g\ g'$

**shows**  $\exists nid' . (g\ m\ p\ h \vdash nid \rightsquigarrow nid') \longrightarrow (g'\ m\ p\ h \vdash nid \rightsquigarrow nid')$   
 $\langle proof \rangle$

Prove that the individual conditional elimination rules are correct with respect to finding a bisimulation between the unoptimized and optimized graphs.

**lemma** *ConditionalEliminationStepProofBisimulation*:

**assumes**  $wf$ : *wf-graph*  $g \wedge wf\text{-stamp}\ g\ stamps \wedge wf\text{-values}\ g$   
**assumes**  $nid$ :  $nid \in ids\ g$   
**assumes**  $conds$ -*valid*:  $\forall c \in conds . \exists v. ([m, p] \vdash c \mapsto v) \wedge val\text{-to}\text{-bool}\ v$   
**assumes**  $ce$ : *ConditionalEliminationStep*  $conds\ stamps\ nid\ g\ g'$   
**assumes**  $gstep$ :  $\exists h\ nid' . (g, p \vdash (nid, m, h) \rightarrow (nid', m, h))$

**shows**  $nid \mid g \sim g'$   
 $\langle proof \rangle$

**experiment begin**

**lemma** *inverse-succ*:

$\forall n' \in (succ\ g\ n). n \in ids\ g \longrightarrow n \in (predecessors\ g\ n')$   
 $\langle proof \rangle$

**lemma** *sequential-successors*:

**assumes**  $is$ -*sequential-node*  $n$   
**shows**  $successors\text{-of}\ n \neq []$   
 $\langle proof \rangle$

**lemma** *nid'-succ*:

**assumes**  $nid \in ids\ g$   
**assumes**  $\neg(is\text{-AbstractEndNode}\ (kind\ g\ nid0))$   
**assumes**  $g, p \vdash (nid0, m0, h0) \rightarrow (nid, m, h)$   
**shows**  $nid \in succ\ g\ nid0$   
 $\langle proof \rangle$

**lemma** *nid'-pred*:

**assumes**  $nid \in ids\ g$   
**assumes**  $\neg(is\text{-AbstractEndNode}\ (kind\ g\ nid0))$   
**assumes**  $g, p \vdash (nid0, m0, h0) \rightarrow (nid, m, h)$   
**shows**  $nid0 \in predecessors\ g\ nid$

*<proof>*

**definition** *wf-pred*:

$wf\text{-}pred\ g = (\forall n \in ids\ g. card\ (predecessors\ g\ n) = 1)$

**lemma**

**assumes**  $\neg(is\text{-}AbstractMergeNode\ (kind\ g\ n'))$

**assumes** *wf-pred g*

**shows**  $\exists v. predecessors\ g\ n = \{v\} \wedge pred\ g\ n' = Some\ v$

*<proof>*

**lemma** *inverse-succ1*:

**assumes**  $\neg(is\text{-}AbstractEndNode\ (kind\ g\ n'))$

**assumes** *wf-pred g*

**shows**  $\forall n' \in (succ\ g\ n). n \in ids\ g \longrightarrow Some\ n = (pred\ g\ n')$

*<proof>*

**lemma** *BeginNodeFlow*:

**assumes**  $g, p \vdash (nid0, m0, h0) \rightarrow (nid, m, h)$

**assumes**  $Some\ ifcond = pred\ g\ nid$

**assumes**  $kind\ g\ ifcond = IfNode\ cond\ t\ f$

**assumes**  $i = find\text{-}index\ nid\ (successors\text{-}of\ (kind\ g\ ifcond))$

**shows**  $i = 0 \iff ([g, m, p] \vdash cond \mapsto v) \wedge val\text{-}to\text{-}bool\ v$

*<proof>*

**end**

**end**

**theory** *CFG*

**imports** *Graph.IRGraph*

**begin**

**datatype** *Block* =

*BasicBlock* (*start-node*: *ID*) (*end-node*: *ID*) |

*NoBlock*

**function** *findEnd* :: *IRGraph*  $\Rightarrow$  *ID*  $\Rightarrow$  *ID list*  $\Rightarrow$  *ID* **where**

*findEnd* *g* *nid* [*next*] = *findEnd* *g* *next* (*successors-of* (*kind* *g* *next*)) |

*findEnd* *g* *nid* *succs* = *nid*

*<proof>* **termination** *<proof>*

**function** *findStart* :: *IRGraph*  $\Rightarrow$  *ID*  $\Rightarrow$  *ID list*  $\Rightarrow$  *ID* **where**

*findStart* *g* *nid* [*pred*] =



```

    (if is-AbstractBeginNode (kind g nid) then
      nid
    else
      (findStart g pred (sorted-list-of-set (predecessors g nid)))) |
  findStart g nid preds = nid
  ⟨proof⟩ termination ⟨proof⟩

fun blockOf :: IRGraph ⇒ ID ⇒ Block where
  blockOf g nid = (
    let end = (findEnd g nid (sorted-list-of-set (succ g nid))) in
    let start = (findStart g nid (sorted-list-of-set (predecessors g nid))) in
    if (start = end ∧ start = nid) then NoBlock else
      BasicBlock start end
  )

fun succ-from-end :: IRGraph ⇒ ID ⇒ IRNode ⇒ Block set where
  succ-from-end g e EndNode = {blockOf g (any-usage g e)} |
  succ-from-end g e (IfNode c tb fb) = {blockOf g tb, blockOf g fb} |
  succ-from-end g e (LoopEndNode begin) = {blockOf g begin} |
  succ-from-end g e - = (if (is-AbstractEndNode (kind g e))
    then (set (map (blockOf g) (successors-of (kind g e))))
    else {})

fun succ :: IRGraph ⇒ Block ⇒ Block set where
  succ g (BasicBlock start end) = succ-from-end g end (kind g end) |
  succ g - = {}

fun register-by-pred :: IRGraph ⇒ ID ⇒ Block option where
  register-by-pred g nid = (
    case kind g (end-node (blockOf g nid)) of
    (IfNode c tb fb) ⇒ Some (blockOf g nid) |
    k ⇒ (if (is-AbstractEndNode k) then Some (blockOf g nid) else None)
  )

fun pred-from-start :: IRGraph ⇒ ID ⇒ IRNode ⇒ Block set where
  pred-from-start g s (MergeNode ends -) = set (map (blockOf g) ends) |
  pred-from-start g s (LoopBeginNode ends - -) = set (map (blockOf g) ends) |
  pred-from-start g s (LoopEndNode begin) = {blockOf g begin} |
  pred-from-start g s - = set (List.map-filter (register-by-pred g) (sorted-list-of-set
    (predecessors g s)))

fun pred :: IRGraph ⇒ Block ⇒ Block set where
  pred g (BasicBlock start end) = pred-from-start g start (kind g start) |
  pred g - = {}

inductive dominates :: IRGraph ⇒ Block ⇒ Block ⇒ bool (- ⊢ - ≥ - 20) where
  [[(d = n) ∨ ((pred g n ≠ {}) ∧ (∀ p ∈ pred g n . (g ⊢ d ≥ p)))] ⇒ dominates
  g d n
code-pred [show-modes] dominates ⟨proof⟩

```

**inductive** *postdominates* :: *IRGraph*  $\Rightarrow$  *Block*  $\Rightarrow$  *Block*  $\Rightarrow$  *bool* (-  $\vdash$  -  $\leq\leq$  - 20)  
**where**  
 $\llbracket (z = n) \vee ((\text{succ } g \ n \neq \{\}) \wedge (\forall s \in \text{succ } g \ n . (g \vdash z \leq\leq s))) \rrbracket \Longrightarrow \text{postdominates } g \ z \ n$   
**code-pred** [*show-modes*] *postdominates*  $\langle$ *proof* $\rangle$

**inductive** *strictly-dominates* :: *IRGraph*  $\Rightarrow$  *Block*  $\Rightarrow$  *Block*  $\Rightarrow$  *bool* (-  $\vdash$  -  $\gg$  - 20) **where**  
 $\llbracket (g \vdash d \geq\geq n); (d \neq n) \rrbracket \Longrightarrow \text{strictly-dominates } g \ d \ n$   
**code-pred** [*show-modes*] *strictly-dominates*  $\langle$ *proof* $\rangle$

**inductive** *strictly-postdominates* :: *IRGraph*  $\Rightarrow$  *Block*  $\Rightarrow$  *Block*  $\Rightarrow$  *bool* (-  $\vdash$  -  $\ll$  - 20) **where**  
 $\llbracket (g \vdash d \leq\leq n); (d \neq n) \rrbracket \Longrightarrow \text{strictly-postdominates } g \ d \ n$   
**code-pred** [*show-modes*] *strictly-postdominates*  $\langle$ *proof* $\rangle$

**lemma** *pred*  $g \ nid = \{\}$   $\longrightarrow \neg(\exists d . (d \neq nid) \wedge (g \vdash d \geq\geq nid))$   
 $\langle$ *proof* $\rangle$

**lemma** *succ*  $g \ nid = \{\}$   $\longrightarrow \neg(\exists d . (d \neq nid) \wedge (g \vdash d \leq\leq nid))$   
 $\langle$ *proof* $\rangle$

**lemma** *pred*  $g \ nid = \{\}$   $\longrightarrow \neg(\exists d . (g \vdash d \gg nid))$   
 $\langle$ *proof* $\rangle$

**lemma** *succ*  $g \ nid = \{\}$   $\longrightarrow \neg(\exists d . (g \vdash d \ll nid))$   
 $\langle$ *proof* $\rangle$

**inductive** *wf-cfg* :: *IRGraph*  $\Rightarrow$  *bool* **where**  
 $\llbracket \forall nid \in \text{ids } g . (\text{blockOf } g \ nid \neq \text{NoBlock}) \longrightarrow (g \vdash (\text{blockOf } g \ 0) \geq\geq (\text{blockOf } g \ nid)) \rrbracket$   
 $\Longrightarrow \text{wf-cfg } g$   
**code-pred** [*show-modes*] *wf-cfg*  $\langle$ *proof* $\rangle$

**inductive** *immediately-dominates* :: *IRGraph*  $\Rightarrow$  *Block*  $\Rightarrow$  *Block*  $\Rightarrow$  *bool* (-  $\vdash$  - *idom* - 20) **where**  
 $\llbracket (g \vdash d \gg n); (\forall w \in \text{ids } g . (g \vdash (\text{blockOf } g \ w) \gg n) \longrightarrow (g \vdash (\text{blockOf } g \ w) \geq\geq d)) \rrbracket \Longrightarrow \text{immediately-dominates } g \ d \ n$   
**code-pred** [*show-modes*] *immediately-dominates*  $\langle$ *proof* $\rangle$

**definition** *simple-if* :: *IRGraph* **where**  
*simple-if* = *irgraph* [  
(0, *StartNode* *None* 2, *VoidStamp*),  
(1, *ParameterNode* 0, *default-stamp*),  
(2, *IfNode* 1 3 4, *VoidStamp*),  
(3, *BeginNode* 5, *VoidStamp*),  
(4, *BeginNode* 6, *VoidStamp*),  
(5, *EndNode*, *VoidStamp*),

```

(6, EndNode, VoidStamp),
(7, ParameterNode 1, default-stamp),
(8, ParameterNode 2, default-stamp),
(9, AddNode 7 8, default-stamp),
(10, MergeNode [5,6] None 12, VoidStamp),
(11, ValuePhiNode 11 [9,7] 10, default-stamp),
(12, ReturnNode (Some 11) None, default-stamp)
]

```

**value** *wf-cfg simple-if*

```

value simple-if ⊢ blockOf simple-if 0 ≥≥ blockOf simple-if 0
value simple-if ⊢ blockOf simple-if 0 ≥≥ blockOf simple-if 3
value simple-if ⊢ blockOf simple-if 0 ≥≥ blockOf simple-if 4
value simple-if ⊢ blockOf simple-if 0 ≥≥ blockOf simple-if 12

```

```

value simple-if ⊢ blockOf simple-if 3 ≥≥ blockOf simple-if 0
value simple-if ⊢ blockOf simple-if 3 ≥≥ blockOf simple-if 3
value simple-if ⊢ blockOf simple-if 3 ≥≥ blockOf simple-if 4
value simple-if ⊢ blockOf simple-if 3 ≥≥ blockOf simple-if 12

```

```

value simple-if ⊢ blockOf simple-if 4 ≥≥ blockOf simple-if 0
value simple-if ⊢ blockOf simple-if 4 ≥≥ blockOf simple-if 3
value simple-if ⊢ blockOf simple-if 4 ≥≥ blockOf simple-if 4
value simple-if ⊢ blockOf simple-if 4 ≥≥ blockOf simple-if 12

```

```

value simple-if ⊢ blockOf simple-if 12 ≥≥ blockOf simple-if 0
value simple-if ⊢ blockOf simple-if 12 ≥≥ blockOf simple-if 3
value simple-if ⊢ blockOf simple-if 12 ≥≥ blockOf simple-if 4
value simple-if ⊢ blockOf simple-if 12 ≥≥ blockOf simple-if 12

```

```

value simple-if ⊢ blockOf simple-if 0 ≤≤ blockOf simple-if 0
value simple-if ⊢ blockOf simple-if 0 ≤≤ blockOf simple-if 3
value simple-if ⊢ blockOf simple-if 0 ≤≤ blockOf simple-if 4
value simple-if ⊢ blockOf simple-if 0 ≤≤ blockOf simple-if 12

```

```

value simple-if ⊢ blockOf simple-if 3 ≤≤ blockOf simple-if 0
value simple-if ⊢ blockOf simple-if 3 ≤≤ blockOf simple-if 3
value simple-if ⊢ blockOf simple-if 3 ≤≤ blockOf simple-if 4

```

**value** *simple-if*  $\vdash$  *blockOf simple-if 3*  $\leq\leq$  *blockOf simple-if 12*

**value** *simple-if*  $\vdash$  *blockOf simple-if 4*  $\leq\leq$  *blockOf simple-if 0*  
**value** *simple-if*  $\vdash$  *blockOf simple-if 4*  $\leq\leq$  *blockOf simple-if 3*  
**value** *simple-if*  $\vdash$  *blockOf simple-if 4*  $\leq\leq$  *blockOf simple-if 4*  
**value** *simple-if*  $\vdash$  *blockOf simple-if 4*  $\leq\leq$  *blockOf simple-if 12*

**value** *simple-if*  $\vdash$  *blockOf simple-if 12*  $\leq\leq$  *blockOf simple-if 0*  
**value** *simple-if*  $\vdash$  *blockOf simple-if 12*  $\leq\leq$  *blockOf simple-if 3*  
**value** *simple-if*  $\vdash$  *blockOf simple-if 12*  $\leq\leq$  *blockOf simple-if 4*  
**value** *simple-if*  $\vdash$  *blockOf simple-if 12*  $\leq\leq$  *blockOf simple-if 12*

**value** *blockOf simple-if 0*  
**value** *blockOf simple-if 1*  
**value** *blockOf simple-if 2*  
**value** *blockOf simple-if 3*  
**value** *blockOf simple-if 4*  
**value** *blockOf simple-if 5*  
**value** *blockOf simple-if 6*  
**value** *blockOf simple-if 7*  
**value** *blockOf simple-if 8*  
**value** *blockOf simple-if 9*  
**value** *blockOf simple-if 10*  
**value** *blockOf simple-if 11*  
**value** *blockOf simple-if 12*

**value** *pred simple-if (blockOf simple-if 0)*  
**value** *succ simple-if (blockOf simple-if 0)*

**value** *pred simple-if (blockOf simple-if 3)*  
**value** *succ simple-if (blockOf simple-if 3)*

**value** *pred simple-if (blockOf simple-if 4)*  
**value** *succ simple-if (blockOf simple-if 4)*

**value** *pred simple-if (blockOf simple-if 10)*  
**value** *succ simple-if (blockOf simple-if 10)*

**definition** *ConditionalEliminationTest1-test1Snippet-initial* :: *IRGraph* **where**

*ConditionalEliminationTest1-test1Snippet-initial* = *irgraph* [  
  (0, (*StartNode* (Some 2) 7), *VoidStamp*),  
  (1, (*ParameterNode* 0), *IntegerStamp* 32 (-2147483648) (2147483647)),  
  (2, (*FrameState* [] None None None), *IllegalStamp*),  
  (3, (*ConstantNode* (*IntVal* 32 (0))), *IntegerStamp* 32 (0) (0)),

(4, (IntegerEqualsNode 1 3), VoidStamp),  
 (5, (BeginNode 39), VoidStamp),  
 (6, (BeginNode 12), VoidStamp),  
 (7, (IfNode 4 6 5), VoidStamp),  
 (8, (ConstantNode (IntVal 32 (5))), IntegerStamp 32 (5) (5)),  
 (9, (IntegerEqualsNode 1 8), VoidStamp),  
 (10, (BeginNode 16), VoidStamp),  
 (11, (BeginNode 14), VoidStamp),  
 (12, (IfNode 9 11 10), VoidStamp),  
 (13, (ConstantNode (IntVal 32 (100))), IntegerStamp 32 (100) (100)),  
 (14, (StoreFieldNode 14 "org.graalvm.compiler.core.test.ConditionalEliminationTestBase::sink2"  
 13 (Some 15) None 18), VoidStamp),  
 (15, (FrameState [] None None None), IllegalStamp),  
 (16, (EndNode), VoidStamp),  
 (17, (MergeNode [16, 18] (Some 19) 24), VoidStamp),  
 (18, (EndNode), VoidStamp),  
 (19, (FrameState [] None None None), IllegalStamp),  
 (20, (ConstantNode (IntVal 32 (101))), IntegerStamp 32 (101) (101)),  
 (21, (IntegerLessThanNode 1 20), VoidStamp),  
 (22, (BeginNode 30), VoidStamp),  
 (23, (BeginNode 25), VoidStamp),  
 (24, (IfNode 21 23 22), VoidStamp),  
 (25, (EndNode), VoidStamp),  
 (26, (MergeNode [25, 27, 34] (Some 35) 43), VoidStamp),  
 (27, (EndNode), VoidStamp),  
 (28, (BeginNode 32), VoidStamp),  
 (29, (BeginNode 27), VoidStamp),  
 (30, (IfNode 4 28 29), VoidStamp),  
 (31, (ConstantNode (IntVal 32 (200))), IntegerStamp 32 (200) (200)),  
 (32, (StoreFieldNode 32 "org.graalvm.compiler.core.test.ConditionalEliminationTest1::sink3"  
 31 (Some 33) None 34), VoidStamp),  
 (33, (FrameState [] None None None), IllegalStamp),  
 (34, (EndNode), VoidStamp),  
 (35, (FrameState [] None None None), IllegalStamp),  
 (36, (ConstantNode (IntVal 32 (2))), IntegerStamp 32 (2) (2)),  
 (37, (IntegerEqualsNode 1 36), VoidStamp),  
 (38, (BeginNode 45), VoidStamp),  
 (39, (EndNode), VoidStamp),  
 (40, (MergeNode [39, 41, 47] (Some 48) 49), VoidStamp),  
 (41, (EndNode), VoidStamp),  
 (42, (BeginNode 41), VoidStamp),  
 (43, (IfNode 37 42 38), VoidStamp),  
 (44, (ConstantNode (IntVal 32 (1))), IntegerStamp 32 (1) (1)),  
 (45, (StoreFieldNode 45 "org.graalvm.compiler.core.test.ConditionalEliminationTestBase::sink1"  
 44 (Some 46) None 47), VoidStamp),  
 (46, (FrameState [] None None None), IllegalStamp),  
 (47, (EndNode), VoidStamp),  
 (48, (FrameState [] None None None), IllegalStamp),  
 (49, (StoreFieldNode 49 "org.graalvm.compiler.core.test.ConditionalEliminationTestBase::sink0"

```

3 (Some 50) None 51), VoidStamp),
  (50, (FrameState [] None None None), IllegalStamp),
  (51, (ReturnNode None None), VoidStamp)
]

value blockOf ConditionalEliminationTest1-test1Snippet-initial 0
value blockOf ConditionalEliminationTest1-test1Snippet-initial 7

value blockOf ConditionalEliminationTest1-test1Snippet-initial 6
value blockOf ConditionalEliminationTest1-test1Snippet-initial 12

value blockOf ConditionalEliminationTest1-test1Snippet-initial 11
value blockOf ConditionalEliminationTest1-test1Snippet-initial 14
value blockOf ConditionalEliminationTest1-test1Snippet-initial 18

value blockOf ConditionalEliminationTest1-test1Snippet-initial 10
value blockOf ConditionalEliminationTest1-test1Snippet-initial 16

value blockOf ConditionalEliminationTest1-test1Snippet-initial 17
value blockOf ConditionalEliminationTest1-test1Snippet-initial 24

value blockOf ConditionalEliminationTest1-test1Snippet-initial 23
value blockOf ConditionalEliminationTest1-test1Snippet-initial 25

value blockOf ConditionalEliminationTest1-test1Snippet-initial 22
value blockOf ConditionalEliminationTest1-test1Snippet-initial 30

value blockOf ConditionalEliminationTest1-test1Snippet-initial 28
value blockOf ConditionalEliminationTest1-test1Snippet-initial 32
value blockOf ConditionalEliminationTest1-test1Snippet-initial 34

value blockOf ConditionalEliminationTest1-test1Snippet-initial 29
value blockOf ConditionalEliminationTest1-test1Snippet-initial 27

value blockOf ConditionalEliminationTest1-test1Snippet-initial 26
value blockOf ConditionalEliminationTest1-test1Snippet-initial 43

value blockOf ConditionalEliminationTest1-test1Snippet-initial 42
value blockOf ConditionalEliminationTest1-test1Snippet-initial 41

value blockOf ConditionalEliminationTest1-test1Snippet-initial 38
value blockOf ConditionalEliminationTest1-test1Snippet-initial 45
value blockOf ConditionalEliminationTest1-test1Snippet-initial 47

value blockOf ConditionalEliminationTest1-test1Snippet-initial 5
value blockOf ConditionalEliminationTest1-test1Snippet-initial 39

value blockOf ConditionalEliminationTest1-test1Snippet-initial 40
value blockOf ConditionalEliminationTest1-test1Snippet-initial 49

```



**value** *pred ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 29*)  
**value** *succ ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 29*)

**value** *pred ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 43*)  
**value** *succ ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 43*)

**value** *pred ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 42*)  
**value** *succ ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 42*)

**value** *pred ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 45*)  
**value** *succ ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 45*)

**value** *pred ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 5*)  
**value** *succ ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 5*)

**value** *pred ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 49*)  
**value** *succ ConditionalEliminationTest1-test1Snippet-initial*  
(*blockOf ConditionalEliminationTest1-test1Snippet-initial 49*)

**end**